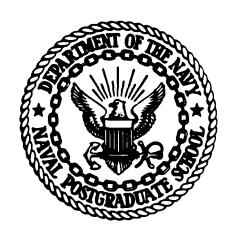


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SUGGESTED IMPROVEMENTS FOR THE JOINT OIL ANALYSIS PROGRAM CORRELATION PROGRAM

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#### INTRODUCTION

The Joint Oil Analysis Program (JOAP) coordinates the Department of Defense (DOD) programs employing spectrometric analyses of used oils for condition monitoring of many types of equipment. Two hundred-odd different oil analysis laboratories provide these spectrometric analyses, the great majority of which are individual-service owned, with the remainder being contract facilities (non DOD). Because of the mobility of equipment, it is quite possible that successive samples of used oil, from the same piece of equipment, may not be analysed on the same instrument. For this reason, and numerous others, it is highly desirable that the same oil sample, when analyzed by different laboratories, should as nearly as possible result in the same contaminant readings.

In the mid 1970's JOAP instituted their "correlation" program, intended to provide information regarding the consistency of readings produced by the spectrometric instruments serving their needs; this program was expected to monitor both internal consistency of repeated readings by the same instrument, as well as consistency from one instrument (or laboratory) to another. The landmark paper discussing this type of problem is by Youden [3], which highlights some empirical observations about instrument-to-instrument testing in general. The procedure Youden describes for checking laboratory to laboratory consistency consists of sending each participating laboratory two "similar" samples of unknown composition; each laboratory receives the same two samples. Each is required to analyze both of the samples (one time) and return the pair of results to a central processing location. If one defines

 $\mathbf{x_i}$  - Analysis result for sample 1, laboratory i

 $\mathbf{y_{f}}$  - Analysis result for sample 2, laboratory i

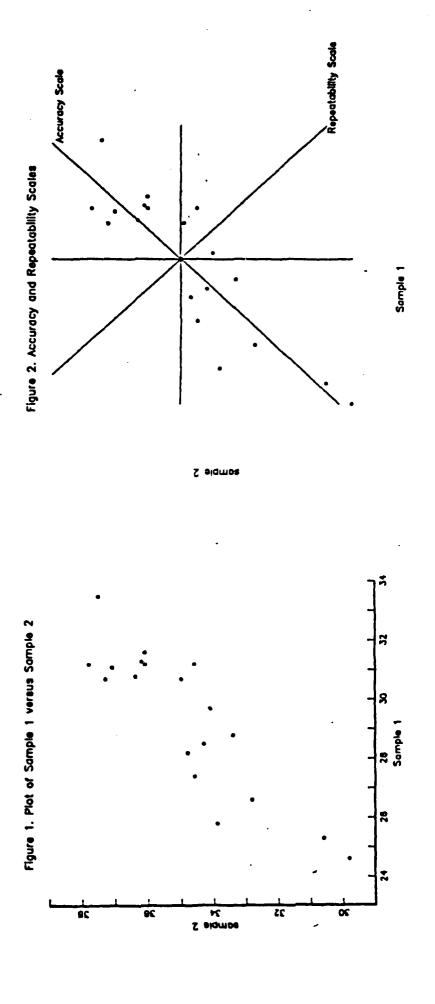
then the n pairs  $(x_i, y_i)$ ,  $i = 1, 2, \ldots$ , n, can be represented as n points in a plane. If one plots these n points, Youden pointed out that the resulting swarm of points almost invariably has the general shape depicted in Figure 1. With coordinate axes at the medians (or means) of the  $x_i$  and  $y_i$  values (as in Figure 2), the preponderance of points will typically fall in the first and third quadrants, with relatively few in the second and fourth. This would necessarily follow in a situation in which a laboratory tends to get either high; readings or low readings, for both of the two samples; if we were to draw in a 45° line and project the points onto this line, the resulting scatter of these projected points describes laboratory-to-laboratory variation. This variation

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be called the variation in accuracy of the laboratories). Youden also pointed out that one can measure the perpendicular distance of each point from this 45° line (i. e., also project the original points onto the normal to the 45° line) to measure the "precision" (or repeatability) of a given laboratory; scatter in this direction should be mainly due to the ability of an individual laboratory to reproduce its own results. Youden suggested that limits defining acceptable laboratory performance can be constructed from the scatter or variation observed in these two directions.

#### THE CURRENT CORRELATION PROGRAM

The JOAP correlation program was modelled after the interlaboratory type of comparison described by Youden, with some important modifications; the basic computations used in the JOAP correlation program are described in [1]. This program is administered by the JOAP Technical Support Center (TSC), located at the Naval Air Rework Facility, Pensacola, FL. Briefly, the correlation program works as follows: Each JOAP laboratory is sent the same pair of oil samples (actually 2 pairs of samples are used, as described later), each month; the particular concentrations of the elements of interest in these samples vary from month to month and are not known by the participating laboratories. laboratory analyzes the pair of samples it receives (presumably only once) and mails the results back to the TSC. Again, let  $(x_i, y_i)$  represent the two sample readings from laboratory i, for a given element. The procedure described in [1] first determines a "trimmed" mean value for the x's and for the y's, independently. These trimmed means are computed by arranging the given x analyses, say, in order of magnitude, deleting the lowest 20% and the highest 20%, and then averaging the remaining middle 60%. Note that it is quite possible that the x score from laboratory 1 might be trimmed off, while its y score is not; that is, a given laboratory's results may contribute to one trimmed mean and not the other. Note as well that only 60% of the x scores received, and 60% of the y scores, are used to define these trimmed means. Letting  $\mathbf{x}_{T}$  and  $\mathbf{y}_{T}$  represent these trimmed means (for a given element and month), the JOAP correlation procedure locates a new coordinate system at  $(x_T, y_T)$ ; these trimmed means play the role of the medians in Youden's discussion [3].

Rather than constructing a 45° line, as suggested by Youden, reference [1] uses a line of slope S, where S is determined by the trimmed means  $(x_T, y_T)$  and constants  $A_j$ ,  $B_j$  which differ from element to element and are presented in Table 1.

Table 1. Constants used to determine slope

Element	A	В		
Fe	2.0	1		
Ag	1.5	.1		
Al	2.0	.1		
Cr	1.5	.1		
Cu '	1.5	.1		
Mg	1.5	.1		
si	1.9	.14		
Ti	1.5	.1		
Ni	1.5	.1		

The slope S used for a given element is defined by

$$s = (A^2 + B^2y_T^2)^{.5}/(A^2 + B^2x_T^2)^{.5}.$$

If  $\mathbf{x}_T = \mathbf{y}_T$ , this formula gives S = 1 (an angle of 45°); indeed, if the two samples sent to a given laboratory have essentially equal concentrations of a given element (the guidelines call for the two to differ by no more than 15%), then this computed slope will not differ from 1 by a great deal. Reference [1] says this formula is intended to avoid "an error unless the composition of the material being measured is identical in the two samples". It is not clear what this expected error might have been, nor does it appear that the slope used by the procedure will materially differ from 1 as Youden suggested. The pair of readings  $(\mathbf{x}_{\hat{\mathbf{I}}}, \mathbf{y}_{\hat{\mathbf{I}}})$  are then projected onto the line with slope S (giving the accuracy score for the laboratory) and onto the line which is normal to this line with slope S (giving the repeatability score for the laboratory).

The major way in which the computations for the JOAP correlation program differ from the procedure suggested by Youden is in the manner in which accuracy (laboratory variation) and repeatability (variability of repeated analyses by the same laboratory with the same sample) are assessed. Reference [1] mentions a current (1973) laboratory certification program which was designed to assure that each laboratory could meet minimum standard performance criteria. This certification program calls for the laboratory to conduct a sequence of ten separate analyses of a prepared oil standard with known concentration c, say, of a given element. The accuracy index (AI) of the laboratory for

this element is the magnitude of the difference between the known concentration, c, and the average of the laboratory's 10 analyses; the repeatability index (RI) of the laboratory for this element is the sample standard deviation of the 10 analyses, computed in the usual way. The acceptable limit for AI is

$$M = (A^2 + B^2c^2)^{.5}$$

where the A and B values are those given in Table 1 above, for the specific element. The laboratory passes the accuracy certification for this element so long as AI < M; it passes the repeatability certification so long as RI < M/2. Thus the constants given in Table 1 were initially proposed for this certification program, and were undoubtedly derived from some physical model of the way in which a particular type of instrument should behave, based on ten repeated analyses of the same sample.

In the correlation program, the accuracy criterion for a given element is defined to be

$$a = (2A^2 + B^2(x_T^2 + y_T^2))^{.5}$$

and the repeatability criterion is a/2, where the constants A and B again come from Table 1 above. Note that a is in fact the square root of the sum of the squares of the M values for the two samples, with the trimmed means  $\mathbf{x}_T$ ,  $\mathbf{y}_T$  playing the roles of the known concentrations c. It is curious that these same constants should be used in the correlation program, where each of two different samples is to be analyzed one time, not ten, and presumably any type of instrument might be used. Each laboratory then is judged on its accuracy and repeatability performance for each element (each month). If the magnitude of its accuracy score exceeds a, it fails on accuracy, and if the magnitude of its repeatability score exceeds a/2, it fails repeatability. This way of defining acceptable limits for the two types of scores depends only on the trimmed means (and the constants A and B) and in no way on the actual scatter of the observed data themselves, contrary to Youden's suggestion. It also leads to quite erratic behavior, in a certain sense, which will be explored below.

It was mentioned earlier that the correlation program actually sends two pairs of samples to each laboratory each month. One pair of samples is prepared by the TSC in new oil, using organo-metallic concentrates with added sulfonate;

it is possible to control the contamination levels of all elements of interest fairly well with these samples. This pair of samples is referred to as "synthesized" samples. In addition to the pair of synthesized samples, the TSC also sends each laboratory a pair of used engine oil samples. These are made from used contaminated oils and, as such, should behave more like actual oil samples the laboratories are expected to analyze daily. It is much more difficult for the TSC to exert control over the contaminant levels in these samples; frequently the same powdered metallic contaminants used for the synthesized samples are added to the used oil samples to adjust the contaminant levels. Thus the correlation program monitors the laboratory performances on both types of samples.

A second dichotomy exists in the correlation program, defined by the physical principle employed by the instrument in measuring concentration. Roughly 80% of the instruments used in JOAP are atomic emission (AE) spectrometers. In these instruments the sample material (the oil) is excited by an electric spark and the spectral lines of the light emitted are used to measure concentrations. The remaining 20% of the instruments used are atomic absorption (AA) spectrometers. In these instruments the sample material is excited by a gas flame, while illuminated by a light of known composition; the amount of the known light absorbed, at specific spectral lines, is used to determine the concentrations in the sample. Because of these different physical bases for measurement, it is well known that the resulting concentration scales are not identical. The correlation program computations are carried out separately for these two types of instrument. Thus a typical JOAP correlation program report contains two major partitions: one describing the behavior of the AE instruments and the other describing the behavior of the AA instruments. Within each of these two, the behaviors for synthesized oil samples and for used oil samples are examined separately, computing the trimmed means, projecting the readings onto "accuracy" and "repeatability" axes, etc., for each element of interest. Although not mentioned in [1], it is undoubtedly true that the constants A and B in table 1, used in defining the accuracy limit a, are derived from a theoretical model of the behavior of a particular atomic emission instrument; nevertheless, the same constants are employed with the AA instruments. At the present time, the same 9 elements are monitored for both types of instrument: iron (Fe), silver (Ag), aluminum (Al), chromium (Cr), copper (Cu), magnesium (Mg), silicon (Si), titanium (Ti), and nickel (Ni).

The correlation program summarizes the monthly behavior of each participating JOAP instrument by a single score, combining behavior over the synthesized and used oil samples. This score is arrived at by subtracting from 100 a certain number of points for each element that the instrument fails to pass (because of its accuracy result or its repeatability result or both) for each sample type for each month. Table 2 presents the number of points lost for each element.

Table 2. Number of points lost for failing accuracy and/or repeatability, either sample type.

Element	Fe	Ag	Al	Cr	Cu	Mg	Si	Ti	Ni	
Points	9	6	4	4	9	4	4	6	4	

Thus, if laboratory 1, say, had failed accuracy for Fe, both accuracy and repeatability for Cr, with synthetic samples, and only failed Cr with used samples, its monthly score would be 100 - 9 - 4 - 4 = 83. If laboratory 2 failed accuracy for Si and Ti, synthetic samples, and both accuracy and repeatability for Cu, used samples, its score would be 100 - 4 - 6 - 9 = 81. If a laboratory fails either accuracy or repeatability for every element, for both types of samples, notice its score would be 0. These monthly scores are used in the correlation program to track laboratory performance over time. The laboratories' 6 month average score is computed and used for certification of the laboratory. If this 6 month average score is below 80 for three consecutive months, the laboratory may be decertified; if the 6 month average score lies between 80 and 90 for 3 consecutive months, the laboratory is provisionally certified. For all other cases the laboratory is continued to be certified.

As mentioned earlier, the acceptable limits for accuracy scores and repeatability scores depend only on the trimmed means  $\mathbf{x}_T$ ,  $\mathbf{y}_T$  and the appropriate constants from Table 1; they do not depend on the actual scatter of the accuracy or repeatability scores themselves. This causes both the accuracy and the repeatability limits, which define the acceptable values, to jump around a great deal, in terms of the number of standard deviations they represent (away from the means, which are 0). Tables 3, 4, 5, and 6 illustrate this phenomenon for the correlation data collected for August, 1986. These tables summarize the number of instruments of the two types that submitted analysis re-

Table 3. August, 1986, summary of correlation scores.

Correlation Scores Used Oils

Atomic Emission August 1986

ı		Accuracy			Repeatability			
,	Limit	#StDev -	#Fail	Limit	#StDev	#Fail	of Labs	
Fe	3.1405	3.2027	10	1.5703	3.8917	0	183	
Ag	2.1213	17.2925	0	1.0607	8.5170	0	183	
Al	2.8284	52.0000	3	1.4142	26.0000	0	183	
cr	2.1215	3.2357	0	1.0607	6.4067	0	183	
Cu	2.5010	3.1894	11	1.2505	3.8719	0	183	
Mg	2.7776	1.9751	32	1.3888	1.9253	3	183	
si	2.8164	3.4913	5	1.4082	3.1023	6	183	
Ti	2.1236	3.0166	1	1.0618	4.9184	0	183	
Mo	2.1291	2.5732	10	1.0646	2.5786	1	182	
Ni	2.1228	3.0808	1	1.0614	4.4233	0	183	

Table 4. August, 1986, summary of correlation scores.

Correlation Scores
Synthetic Oils

Atomic Emission August 1986

	Į	Accuracy	ļ	Repea	Number		
,	Limit	#StDev	#Fail	Limit	#StDev	#Fail	of Labs
Fe	5.5883	3.9480	4	2.7941	3.0941	0	183
Ag	3.0275	2.1700	12	1.5137	1.7183	3	183
Αĺ	5.9450	3.7681	7	2.9725	3.2787	0	183
Cr	2.9595	2.9511	4	1.4798	3.2800	1	183
Cu	3.8884	2.4030	10	1.9442	1.4166	5	183
Mg	8.5668	2.8151	20	4.2834	2.2899	4	183
Si	15.9339	4.8487	5	7.9670	4.6366	0	183
Ti	3.8749	2.6859	9	1.9374	2.8417	1	183
Mo	2.6922	1.8139	9	1.3461	2.5420	1	182
Ni	3.2006	3.6200	2	1.6003	3.1763	0	183

Table 5. August, 1986, summary of correlation scores.

## Correlation Scores Used Oils

Atomic Absorption August 1986

1		Accuracy			Repeatability			
	Limit	#StDev	#Fail	Limit	#StDev	#Fail	of Labs	
Fe	2.9089	2.3724	6	1.4545	3.8233	0	37	
λg	2.1213	10.5623	1	1.0607	5.2812	0	37	
Al	2.8292	4 - 0772	0	1.4146	8.4949	0	37	
Cr	2.1213	- 3.5942	0	1.0607	∞	0	37	
Cu	2.2341	3.6256	0	1.1171	3.3077	0	37	
Mg	2.2976	2.2609	7	1.1488	3.3621	0	37	
si	2.7267	2.6933	11	1.3634	8.6240	3	31	
Ti	2.1216	3.5411	5	1.0608	∞	0	32	
Mo	2.1213	<b>∞</b>	1	1.0607	<b>∞</b>	1	14	
Ni	2.1220	3.1295	5	1.0610	∞	1	35 <sup>.</sup>	

Table 6. August, 1986, summary of correlation scores.

Correlation Scores
Synthetic Oils

Atomic Absorption August 1986

	1	Accuracy	1	Repea	Number		
	Limit	#StDev	#Fail	Limit	#StDev	#Fail	
Fe	5.4000	2.2351	7	2.7000	3.1609	- 0	37
Ag	2.9638	2.1089	8	1.4819	3.3577	1	37
Αĺ	5.5213	2.0836	7	2.7607	2.7949	2	37
(	2.7500	1.6746	13	1.3750	3.1476	1	37
Cu	3.6054	3.5904	4	1.8027	3.9123	1	37
Mg	8.9353	1.9332	14	4.4676	3.7263	3	37
Si	15.1153	2.3767	11	7.5577	2.9521	0	31
Ti	3.8108	1.9218	12	1.9054	2.6810	0	32
Mo	2.7003	1.6592	4	1.3501	3.0514	1	14
Ni	3.0897	2.2816	9	1.5448	3.0922	2	35

sults for the various elements, as well as the computed acceptable limits for accuracy and repeatability, for both types of samples. In addition, the actual standard deviations of the scores have been computed. Recall that 40% of the data was trimmed, for both the x (Sample one) scores and for the y (Sample two) scores, in locating the origin for the accuracy and repeatability axes. standard deviations used in this discussion are computed from the readings provided by those instruments which were kept after the trimming, for one or both of the two samples. In every case, the means for these values are essentially 0, so the standard deviations were computed about 0. The column labelled #StdDev gives the ratio of the limit (given in the column labelled Limit) for the given variable divided by this computed standard deviation. Note that the number of standard deviations which the limits represent vary quite widely from element to element, especially for the used samples (for both types of instrument). They also vary quite widely from month to month; Appendix A presents the same type of data for one additional month, January, 1985, the only other month for which we have all the necessary data available in electronic form for AE instruments. For AA instruments, an additional 18 months of data has been available to us; although not included with this report, there is tremendous variation in the ratio of the limit divided by the standard deviation from element to element, and from month to month for the same element. There does not seem to be any logical reason that one would like this type of ratio to vary in this way. seem to indicate that there is a wide variablity in the ease with which a laboratory could meet the accuracy and repeatability requirements from month to month.

The instruments read out concentration values to the nearest .1; these values are reported to the TSC. The data entered into the computer for the correlation program computations is rounded to the closest integer, causing a large number of pairs of analyses to be identical. This phenomenon in turn can lead to all of the nontrimmed accuracy scores (or equally well the repeatability scores), which are used to compute the standard deviation, being equal; such a standard deviation then is 0 and the ratio of the computed limit to such a standard deviation is of course undefined. This situation is labelled by the symbol  $\infty$  in the #StdDev column (see e. g., Table 5, Cr, repeatability). If the data were entered with full accuracy (including tenths) it is expected this phenomenon will not occur very frequently.

#### SUGGESTED IMPROVEMENTS

The paper by Youden, discussed earlier, led to the publication of a number of additional contributions to the literature about interlaboratory comparisons. An interesting paper [2] was published by Mandel and Lashof, giving interpretations and more mathematical discussions of Youden's ideas, one year after [1] discussed the JOAP correlation program as now implemented. Among other things, Mandel and Lashof give some models which make the scatter of points mentioned by Youden seem natural, as well as changes to these models which could reasonably lead to quite different looking plots. They suggest that the bivariate normal distribution provides a good model for the original pairs of sample readings  $(x_i, y_i)$ ; if one lets  $\bar{x}$ ,  $\bar{y}$  represent the means of the observed pairs, then the pairs  $(x_i - \bar{x}, y_i - \bar{y})$  will be bivariate random variables with means equal to zero. One can then use principal components to find the direction of the axis which includes the greatest variability; for some simple reasonable types of structures this direction turns out to be the line with slope 1, the phenomenon pointed out by Youden. The orthogonal direction is the one with the least variability, and is free of effects of different instruments under a standard type of linear model; for the types of samples used in the JOAP correlation program, it would appear that the simple type of linear model they discuss should be appropriate. The following discussion incorporates some of the ideas and suggestions made by Mandel and Lashof.

For a given month, for a given element, and type of instrument, let  $(x_i, y_i)$  represent the observed pairs of analyses received, i = 1, 2, ..., n, where n is the number of instruments. Let us assume that

$$x_i - \mu_1 + L_i + e_i,$$

$$y_i - \mu_2 + L_i + f_i$$

for  $i=1, 2, \ldots, n$ .  $\mu_1$  and  $\mu_2$  represent the "true" contents of samples 1 and 2, respectively;  $L_i$  is meant to represent a laboratory effect that is constant for both  $x_i$  and  $y_i$ , the two sample readings received from the same laboratory.  $e_i$  and  $f_i$  represent independent random measurement errors (or noise) for the two analyses. It seems quite reasonable to assume that the  $e_i$  and  $f_i$  values are independent and normally distributed with the same variance; as Mandel and Lashof suggest, one can also assume that the laboratory effects,  $L_i$ , are normally distributed with

buted. It follows then that

$$X_i - x_i - \bar{x} - L_i - \bar{L} + e_i - \bar{e}$$

$$Y_i = y_i - \bar{y} - L_i - L + f_i - \bar{f}$$

that is, the pairs  $(X_i, Y_i)$  do not depend on the true contents  $\mu_1$ ,  $\mu_2$ , but only on the laboratory effects and the measurement errors. The projection of  $(X_i, Y_i)$  onto the 45° line, times the square root of two, then is

$$X_i + Y_i = 2(L_i - \hat{L}) + (e_i - \hat{e}) + (f_i - \hat{f}).$$

Note that this sum is affected by the laboratory effects as well as the measurement errors. The projection of  $(X_{\dot{1}}, Y_{\dot{1}})$  onto the line normal to the 45° line, times the square root of two, is

$$X_{i} - Y_{i} = (e_{i} - \hat{e}) - (f_{i} - \hat{f}),$$

a quantity which depends only on the measurement errors, and not the laboratory effects (or the true contents). With this simple type of additive model it indeed turns out that the projections on the 45° line give a reasonable idea of accuracy, or spread, among the different laboratories and the projections on the orthogonal axis depend only on the measurement errors, or repeatability, of an instrument's readings.

It is not uncommon for "wild" points to occur in using sensitive instruments to make fine measurements; undoubtedly the reason for trimming the data in the correlation program is to remove these effects. While we agree with this general principle (using trimming to remove outliers) it also seems that 40% trimming is very extreme. The idea of independent trimming of the two samples is also not particularly appealing, allowing, as already mentioned, the possibility that an instrument's y-score is trimmed, but its x-score is not. We have made a preliminary investigation into trimming, possible methods for doing it as well as the amount of trimming to apply. It has been pointed out some time ago that the untrimmed means and the trimmed means, using actual correlation data, result in essentially the same value for all elements for all months. That is, because of the number of laboratories participating, and the apparent fact that "wild" points seem to occur symmetricly (some big, some small), the location is

essentially unchanged if one uses untrimmed means instead of trimmed means. Since the current correlation program computations depend only on the trimmed means (and constants), one would get the same scores and results if one used the full set of raw data with no trimming. If, however, one wants to use the observed scatter or spread on the accuracy and repeatability axes to determine limits for acceptable behavior, "wild" points could seriously inflate the results; thus we are in favor of applying some trimming before establishing limits for accuracy and repeatability. We are also in favor of the philosophy of bivariate trimming: if a laboratory is trimmed on the x-scale it is also necessarily trimmed on the y-scale.

There are many different ways to implement bivariate trimming. If one adopts the suggestions of normality put forth by Mandel and Lashof, it would seem natural to use the constant contours of the bivariate normal density function to accomplish the trimming. Letting  $(X_i, Y_i)$  be as defined above, this means computing and inverting a 2 by 2 matrix (details are given in Appendix B), and then evaluating quadratic forms (locating the contours which contain the observed points). Those points most distant from the origin are the candidates for trimming. Figure 3 presents a typical scatter of observed sample results, with 3 of the bivariate normal constant contours drawn in. We have applied this type of trimming to the August, 1986, and January, 1985, data available to us; 20%, 10% and 5% trimming were looked at and for these data it appears that 5% contour trimming does a sufficiently good job for both types of instrument. Thus for the 20 points pictured in Figure 3, 5% trimming would delete the single point on the outtermost ellipse.

We recommend that for a period of time (6 months or more) the scores for the correlation program be computed as present and compared with scores developed essentially according to Youden's original suggested procedure for interlaboratory comparisons. Specifically, this second set of scores will employ 5% bivariate trimming (as mentioned above and defined in Appendix B) to guard against "wild" points. The trimmed means  $\bar{x}$ ,  $\bar{y}$  are computed from the remaining observatons and used to define  $X_i = x_i - \bar{x}$  and  $Y_i = y_i - \bar{y}$ , centering at these trimmed means. Then all observed pairs  $(X_i, Y_i)$  will be projected onto the 45° line (accuracy scores) and onto the line normal to this line (repeatability scores). The original untrimmed values are then used to compute standard deviations in each of these two dimensions (see Appendix B), which in turn are used to define the acceptable accuracy and repeatability limits. Any laboratory which has an

Figure 3. Constant Bivariate Normal Contours 32 30 Sample 28 26 0Σ 8Σ 9Σ **γ**Σ 2Σ Sample 2

accuracy score whose magnitude exceeds the accuracy limit fails on accuracy; any laboratory which has a repeatability score whose magnitude exceeds the repeatability limit fails on repeatability.

The number of standard deviations to use in defining the accuracy and repeatability limits is, of course, arbitrary, and can be set at any level desired. We recommend that 3 standard deviations be used at least initially; thus the accuracy limit will be 3s<sub>a</sub> and the repeatability limit will be 3s<sub>r</sub>, where s<sub>a</sub> and s<sub>r</sub> are the computed standard deviations. A rationale for using 3 as a multiplier is given below. This procedure has been applied to both the August, 1986, and the January, 1985 data; tables 7, 8, 9, and 10 present the resulting limits for the January, 1986 data and are comparable to tables 3, ...,6 presented earlier. The computations for January, 1985, are also presented in Appendix A.

In comparing the correlation scores with these proposed scores, perhaps the most apparent difference is the increased number of laboratories which fail on repeatability (and relatively fewer on accuracy). It would appear that the correlation program method of determining the limits for repeatability (which depend only on the 60% trimmed means) does not provide an effective check. Note as well that the AA instruments in general fare much better with the proposed method than they do with the current correlation procedure. As mentioned above, the procedure for determining the limits in the correlation program includes the constants A and B which were undoubtedly derived for an AE instrument and do not perform well for an AA instrument. It is also of interest to compare which particular instruments fail on accuracy and/or repeatability for the correlation program versus the proposed procedure. Appendix A presents this information for the August, 1986, and January, 1985 data.

One rationale for determining the number of standard deviations to use in setting the accuracy and repeatability limits can be defined in terms of the chances of an instrument, which performs correctly, passing both the accuracy and repeatability tests, for all elements, for both used and synthetic samples. Let p represent the probability that a correctly functioning instrument will fail the check, for either accuracy or repeatability (the same value for both). Then the probability that it will pass both accuracy and repeatability, for any element, is  $(1-p)^2$  and the probability it will pass for all nine elements for both used and synthetic samples (assuming independence) is  $(1-p)^{36}$ . Suppose we set this quantity equal to .9; this gives the value for p to be  $1-.9^{1/36}$ , or

Table 7. Proposed method for determining scores.

Proposed Scores Used Oils

Atomic Emission August 1986

1	ì	Accuracy	1	Repea	Number		
	StDev	Limit	#Fail	StDev	Limit	#Fail	of Labs
Fe	1.2649	3.7947	4	.3772	1.1316	2	183
Ag	.4575	1.3725	0	.0000	.0000	5	. 183
Aĺ	.3116	.9347	14	.0000	.0000	15	183
Cr	.6408	1.9223	0	.1054	.3162	10	183
Cu	1.1871	3.5613	6	.2963	.8888	0	183
Mg	1.8946	5.6838	8	.3643	1.0928	3	183
sī	.9489	2.8467	5	.3752	1.1257	8	183
Ti	.7609	2.2826	1 \	.1571	.4712	18	183
Mo	1.0697	3.2091	0	.3541	1.0623	1	182
Ni	.6876	2.0629	2	.2243	.6729	22	183.

Table 8. Proposed method for determining scores.

Proposed Scores Synthetic Oils Atomic Emission August 1986

		Accuracy	1	Repea	Number		
	StDev	Limit	#Fail	StDev	Limit	#Fail	of Labs
Fe	1.9422	5.8265	2	.7871	2.3614	2	183
Ag	1.4674	4.4022	3	.3993	1.1978	6	183
AĬ	2.2865	6.8594	7	.8169	2.4506	1	183
Cr	1.0171	3.0512	3	.4124	1.2371	2	183
Cu	1.6348	4.9044	5	.5399	1.6197	5	183
Mg	4.4506	13.3518	2	1.6082	4.8246	2	183
Si	4.8085	14.4255	5	1.5389	4.6168	2	183
Ti	1.6961	5.0882	4	.5924	1.7772	1	183
Mo	1.4966	4.4898	1 )	.4855	1.4564	1 )	182
Ni	1.0884	3.2651	2	.4284	1.2851	5	183

Table 9. Proposed method for determining scores.

Proposed Scores Used Oils Atomic Absorption August 1986

	1	Accuracy	. [	Repea	Number		
	StDev	Limit <sup>2</sup>	#Fail	StDev	Limit	#Fail	of Labs
Fe	2.1078	6.3234	0	.3357	1.0071	0	37
Ag	.4452	1.3356	1	.2226	.6678	0	37
ΑĬ	.8396	2.5188	0	.1621	.4863	2	37
Cr	.5902	1.7706	0	.0000	.0000	0	37
Cu	.7071	2.1213	0	.3285	.9856	0	37
Mg	2.5244	7.5731	0	.4008	1.2025	0	37
si	3.1192	9.3576	1	.4348	1.3045	3	31
Ti	2.6074	7.8221	1	.1739	.5217	2	32
Mo	9449	2.8347	1	.5669	1.7008	1	14
Ni	1.1114	3.3343	2	.0000	.0000	1	35

Table 10. Proposed method for determining scores.

Proposed Scores Synthetic Oils Atomic Absorption August 1986

	1	Accuracy			Repeatability		
	StDev	Limit	#Fail	StDev	Limit	#Fail	of Labs
Fe	4.1044	12.3132	1	.8962	2.6886	0	37
Αg	2.5229	7.5687	1	.4309	1.2928	1	37
ΑĪ	5.7968	17.3905	2	.9968	2.9905	0	37
Cr	3.0909	9.2727	2	.5297	1.5890	1	37
Cu	2.0240	6.0719	2	.6409	1.9226	1	37
Mg	19.1620	57.4860	2	1.8851	5.6553	2	37
si	35.6186	106.8559	0	2.3484	7.0452	1	3.1
Ti	5.8034	17.4102	2	.7199	2.1597	0	32
Mo	3.3033	9.9100	0	3.6025	10.8075	1	14
Ni	3.9572	11.8716	2	.7932	2.3797	2	35

.9971. With the assumption of normality for the measurement errors for the instruments, and for the variation between instruments, we then require the number of standard deviations that a normal random variable will exceed with probability .00145 (half the value of 1 - .9971 = .0029, since the projected scores can be extreme either positively or negatively). This, in turn, results in a requirement of 2.978 standard deviations, which we have rounded to 3 for a trial of the proposed system.

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This appendix presents additional numerical tables; Tables A1 through A8 present the correlation program data and the proposed program data for the month of January, 1985; the format and information presented is identical to that given earlier in Tables 3 through 10, for August 1986.

Table A1. January, 1985, summary of correlation scores.

Correlation Scores

Atomic Emission
Used Oils

January 1985

	USEC UIL	. 5		January 1705				
	ł	Accuracy			. Repeatability			
	Limit	#StDev	#Fail	Limit	#StDev	#Fail	of Labs	
Fe	3.5071	2.6565	9	1.7535	1.9128	1	180	
Ag	2.1213	3.6428	2	1.0607	9.9782	0	180	
AL	2.8943	2.0591	28	1.4472	3.1567	2	180	
Cr	2,1260	9.1720	1	1.0630	4.6267	0	180	
Cu	3.9245	2.2462	31	1.9623	2.9665	4	180	
Mg	2.1508	3.0324	5	1.0754	4.2906	0	179	
Si	2.7349	3.0866	6	1.3675	2.3458	2	180	
Ti	2.1260	9.9737	3	1.0630	4.9869	1	180	
Mo	2,1306	2.6244	17	1.0653	1.9102	7	176	
Ni	2.1243	3.0390	3	1.0622	3.3791	2	180	

Table A2. January, 1985, summary of correlation scores.

Correlation Scores Atomic Emission
Synthetic Oils January 1985

	1	Accuracy	i	Repea	i	Number			
	Limit	#StDev	#Fail	Limit	#StDev	#Fail	-of Labs		
Fe	10.1637	3.9005	6	5.0819	3.0955	3	180		
Ag	2.6760	2.6029	19	1.3380	3.1013	1	180		
AL	3.6902	2.4123	12	1.8451	2.2952	7	180		
Cr	2.8611	3.5102	2 1	1.4305	2.4633	4	180		
Cu	2.8887	2.9225	5	1.4444	3.0815	0	180		
Mg	4.8431	3.0231	16	2.4215	2.3116	4	179		
Si	26, 1873	4.1619	8	13.0937	3.8006	0	180		
Ti	3.0830	3.0546	19	1.5415	2.5627	4	180		
Mo	2.1234	3.0510	2	1.0617	2.4013	5	176		
Ni	2.5477	3.6203	2	1.2739	2.8681	0	180		

Table A3. January, 1985, summary of correlation scores.

Correlation Scores Atomic Absorption

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January 1985 Used Oils Repeatability Accuracy #Fail #StDev of Labs Limit Limit #StDev #Fail 3.2953 3.0387 1.5194 3.9041 0 40 Fe 39 8.7464 n Ag 2.1213 17.4929 0 1.0607 ΑĬ 2.9019 4.3052 2 1,4510 3.6814 0 39 8.2341 0 39 2.1260 1.0630 Cr 16.4682 1 Cu 2.8149 2.5119 1 1.4074 2.6220 40 2.1299 2.9297 1.0650 39 3,6705 0 Mg 2 23 Si 2.7107 1.7149 6 1.3554 2.2365 2 Ti 2.1214 3.9403 3 1.0607 0 27 1.0607 0 22 2.1213 2 8.7475 2.1216 3.1765 0 1.0608

Table A4. January, 1985, summary of correlation scores.

Correlation Scores Atomic Absorption

January 1985 Synthetic Oils Accuracy Repeatability #Fail #StDev #Fail of Labs Limit #StDev Limit Fe 2.1496 10,4337 2.1215 7 5.2169 2.8423 39 1.4074 0 Ag 2.8148 1.5639 8 AĬ 3.5187 3.7979 1.7594 4.4148 0 39 2.4828 39 1.4257 Cr 2.8514 2.0918 12 40 2.3404 Cu 2.8320 2.8357 2 1.4160 4.9521 2.4760 2.3912 39 Mg 1.8625 8 2.5849 3.0578 23 26.9246 13.4623 81 2 1 27 Ti 3.1099 1.1944 10 1.5549 2.8637 2 1.0630 22 1 2.1260 Mo 2.0737 1.0353 1.2657 2.5315

Table A5. January, 1985, summary of proposed scores.
Proposed Scores Atomic Atomic Emission

	Used Oil			January 1985						
		Accuracy	1	Repes	Number					
	StDev	Limit	#Fail	StDev	Limit	#Fail	of Labs			
Fe	1.4253	4.2760	6	<b>.4200</b>	1.2600	6	180			
PA	.5860	1.7579	2	.0000	.0000	4	180			
AL	2.0972	6.2915	4	.4005	1.2014	5	180			
Cr	.7603	2.2810	1 ]	. 1646	.4938	14	180			
Cu	2.7733	8.3198	4	.5965	1.7896	4	180			
Mg	.9294	2.7882	5	.2163	.6489	22	179			
Si	.8833	2.6500	6	.3574	1.0723	2	180			
Ti	.7462	2.2386	3	. 1405	.4216	14	180			
Mo	1.1451	3.4352	o l	.4113	1.2339	7	176			
Ni	.7389	2.2168	3	.2340	.7021	12	180			

Table Aó. January, 1985, summary of proposed scores.
Proposed Scores Atomic Atomic Emission

	Synthet	ic Mile		January 1985						
	1	Accuracy	1	Repea	Number					
	StDev	Limit	#Fail	StDev	Limit	#Fail	of Labs			
Fe	3.8243	11.4729	5	1.5377	4.6130	5	180			
Ag	1.2390	3.7169	3	.3786	1.1358	2	180			
ΑĬ	1.8333	5.4999	2	.6537	1.9612	7	180			
Cr	.9654	2.8963	2	.5081	1.5242	4	180			
Cu	1.1571	3.4712	- 1	.4341	1.3022	0	180			
Mg	2.3905	7.1716	6	.9018	2.7054	3	179			
Si	9.0119	27.0357	7	3.1042	9.3125	6	180			
Ti	1.3609	4.0826	6	.5680	1.7039	1	180			
Mo	.7933	2.3798	2	.3221	.9662	5	176			
Ni	.8372	2.5117	2	.3957	1.1870	4	180			

Table A7. January, 1985, summary of proposed scores. Proposed Scores Atomic Absorption

	Used Oi	ls		January 1985						
	!	Accuracy	1	Repea	Number					
	StDev	Limit	#Fail	StDev	Limit	#Fail	of Labs			
Fe	1.6809	5.0427	2	.3023	.9069	1	40			
Ag	.4845	1.4534	0	.0000	.0000	1	39			
ΑĬ	1.6989	5.0966	0	.3288	.9864	1	39			
Cr	.7076	2.1228	2	. 1600	.4800	2	39			
Cu	1.1350	3.4049	0	.2786	.8357	1	40			
Mg	.8625	2.5876	1 }	.2822	.8467	0	39			
Si	6.0414	18.1243	2	1.0756	3.2268	1	23			
Ti	1.0539	3.1618	2	.0000	.0000	3	27			
Mo	.6761	2.0284	2	.0000	.0000	2	22			
Ni	.6770	2.0310	ō	.0000	.0000	1	34			

		sed Scores	os, sammer	Atomic Absorption						
	Synthet	ic Oils		January 1985						
	1	Accuracy	1	Repe	etability	·	Number			
	StDev	Limit	#Fail	StDev	Limit	#Fail	of Labs			
Fe	8.4609	25.3826	3	2.6683	8.0049	0	40			
Ag	2.3520	7.0561	1	.5102	1.5306	0	39			
ΑĬ	2.3069	6.9207	2	.4255	1.2764	1	39			
Cr	2.7603	8.2808	o i	.6728	2.0185	0	39			
Cu	1.1571	3.4713	0	.5078	1.5233	2	40			
Mg	5.4262	16.2786	2	1.1346	3.4038	0	39			
Si	18.5882	55.7646	2	5.1979	15.5938	0	23			
Tí	5.0776	15.2329	1	.8288	2.4864	0	27			
Mo	.6761	2.0284	1	.0000	.0000	1	22			
Ni	2.2082	6.6245	1	.5303	1.5910	1	34			

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Tables A9 through A24 present the indices of those laboratories which failed accuracy and/or repeatability, for the current correlation program and for the proposed method of scoring.

Table A9. Indices of Failing Labs
Correlation Scores Atomic Emission
Used Oils August 1986

Fe	Ag	AL	Cr	Cu	Йg	Si	Ti	No	Ni
Acc Rep 9 33 39 55 96 152 177 181 182	Acc Rep	Acc Rep. 11 49 151	Acc Rep	Acc Rep 49 55 118 146 147 148 151 157 164 181 182	Acc Rep 1 24 3 181 9 182 13 15 16 24 33 39 40 55 79 88 90 95 98 112 117 127 131 147 148 152 169 170 171 171 172 175 177 181 182	Acc Rep 9 6 112 33 134 59 155 79 165 112 165	Acc Rep 182	Acc Rep 9 12 11 15 41 49 78 102 118 166 180	Acc Rep 146

Table A10. Indices of Failing Labs
Proposed Scores Atomic Emission
Used Oils August 1986

Fe	Ag	AL	Cr	Cu	Mg	Si	Ti	Мо	Ni
Acc Rep 152 122 177 181 181 182	Acc Rep 2 3 113 121 173	Acc Rep 1 1 1 7 7 7 8 8 11 11 31 31 41 36 44 41 49 44 151 54 161 151 162 161 173 162 181 173	Acc Rep 333 411 49 68 75 124 140 167 174 182	55 118 146	Acc Rep 9 24 15 181 16 182 24 33 148 181 182	Acc Rep 9 12 33 134 59 155 79 165 103 112 165 182	Acc Rep 182 42 67 71 78 82 88 89 92 98 130 134 140 151 153 167 176 177	Acc Rep 12	Acc Rep 146 35 177 36 37 42 59 67 71 77 85 90 98 101 106 122 130 134 143 146 156 163 167 174

Table All. Indices of Failing Labs Correlation Scores Atom Synthetic Oils

Atomic Emission August 1986

Fe	Ag	AL	Cr	Cu	Mg	Si	Ti	Mo	Ni
Acc Rep 94 97 164 173	Acc Rep 23 136 24 140 54 177 88 97 114 134 135 155 164 167 177	Acc Rep 1 78 108 164 175 177 182	Acc Rep 15 177. 33 136 164			114 164	Acc Rep 16 179 78 95 114 134 135 164 173 182		

Table A12. Indices of Failing Labs Proposed Scores Atom Synthetic Oils -

Atomic Emission August 1986

Fe	Ag	AL	Cr	Cu	Mg	Si	Ti	Mo	Ni
Acc Rep 164 140 173 173	114 71	1 177 78 108 164 175		32 32	16 88 173 160	114 140	16 179		Acc Rep 164 82 182 88 132 135 177

Table A13. Indices of Failing Labs

Correlation Scores Used Oils

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Atomic Absorption August 1986

Fe Cr Cu Ti Ni Ag Mg Acc Rep 8 14 15 16 17 24 34 12 9 12 14 3 14 14 12 89 15 10 19 16 17 25 26 31 24 12 11 24 12 18 24 26 26 22 23 24 28 31

Table A14. Indices of Failing Labs

Proposed Scores Used Oils Atomic Absorption August 1986

Fe	Ag	AL	Cr	Cu	Mg	\$ i	Ti	Мо	Ni
Acc Rep	Acc Rep 12	Acc Rep 27 31	Acc Rep	Acc Rep	Acc Rep	Acc Rep 11 8 11 24	Acc Rep 26 9 26	14 14	

Table A15. Indices of Failing Labs

Correlation Scores Synthetic Oils Atomic Absorption August 1986 

Fe	Ag	Al	Cr	Cu	Mg	Si	Ti	Mo	Ni
Acc Rep 8 12 24 26 31 32 34		Acc Rep	Acc Rep	-	Acc Rep				

Table A16. Indices of Failing Labs

Proposed Scores Synthetic Oils Atomic Absorption August 1986

Fe	Ag	AL	Cr	Cu	Mg	Si	Ti	Мо	Ni
Acc Rep 24	Acc Rep 28 28		Acc Rep 17 31 24		Acc Rep 12 7 37 34	20		Acc Rep 14	Acc Rep 23 12 32 32

Table A17. Indices of Failing Labs Correlation Scores Atom Used Oils

Atomic Emission January 1985

fe	Ag	AL	Cr	Cu	Mg	Si	Ti	Mo	Ní
Acc Rep 37 51 51 75 78 108 135 156 172 176		Acc Rep 10 6 12 47 18 23 24 34 35 37 42 45 58 80 88 111 118 124 132 149 154 158 160 161 168 172 174 180	21	Acc Rep 1 130 6 158 12 169 23 172 26 38 42 89 93 100 101 117 124 128 130 133 135 136 140 141 147 154 156 160 164 167 168 172 173 174 176	Acc Rep 23 92 156 157 160	Acc Rep 19 126 23 180 92 126 152 180	Acc Rep 35 79 79 164	Acc Rep 10 1 12 38 37 57 42 61 59 135 76 157 80 160 93 114 126 134 135 138 145 157 168 171	25 25 35 135 161

Table A18. Indices of Failing Labs Proposed Scores Ator Used Oils

Atomic Emission January 1985

Fe	Ag	AL	Cr	Cu	Mg	\$i	Ti	Mo	Ni
Acc Rep 37 17 51 51 108 79 135 89 172 108 176 172	Acc Rep 22 45 161 108 109 168	Acc Rep 118 6 158 23 161 47 172 101 158	Acc Rep 21 32 63 70 118 119 128 141 144 150 155 161 169 172	Acc Rep 42 130 140 158 172 169 176 172	23 1 92 5 156 6	Acc Rep 19 126 23 180 92 126 152 180	Acc Rep 35 5 79 6 164 13 19 27 31 59 64 72 79 140 150 180	Acc Rep 1 38 57 61 135 157 160	Acc Rep 25 19 35 23 161 25 31 49 72 104 135 151 156 168 177

# Table A19. Indices of Failing Labs Correlation Scores Atomic Emission Synthetic Oils January 1985

Fe	Ag	AL	Cr	Cu	Mg	\$i	Tí	Мо	Ni -
Acc Rep 108 50 128 102 170 108 171 173 178	Acc Rep 4 37 23 31 34 38 71 75 76 102 135 148 151 152 154 157 161 172 176 178	Acc Rep 10 71 25 108 41 110 76 124 101 158 102 174 138 180 143 161 170 178 180	154 74 157 108 119 174	Acc Rep 23 118 154 173 178	Acc Rep 2 102 103 31 108 48 138 54 65 79 108 115 127 148 159 168 171 175 178	29	Acc Rep 12 51 31 102 38 108 71 120 77 79 103 108 137 154 155 159 161 164 170 172 173 175 178	114 80 160 89	Acc Rep 12 170

# Table A20. Indices of Failing Labs Proposed Scores- Atomic Emission Synthetic Oils January 1985

Fe '	Ag	AL .	Cr	Cu	Mg	Si	Ti	Мо	Ni
Acc Rep 108 50 128 67 170 102 171 108 178 136	31 37 151 176 176	138 71	154 74 157 108 119	23	Acc Rep 31 102 108 103 115 108 171 175 178	29 29 63 42	79 108 154 161 164 173		12 17

# Table A21. Indices of Failing Labs Correlation Scores Atomic Absorption Used Oils January 1985

Fe	Ag	AL	Cr	Cu	Mg	Si	Ti	Мо	Ni
Acc Rep 19 28 29 37	Acc Rep	Acc Rep 19 37	Acc Rep 36	Acc Rep 9 33		Acc Rep 9 9 10 14 13 14 20 23	Acc Rep 7 10 23	Acc Rep 7 20	Acc Rep

# Table A22. Indices of Failing Labs Proposed Scores Atomic Absorption Used Oils January 1985

	Fe	Ag	AL	Cr	Cu	Mg	Si	Ti	Mo	Ni
Ac 2 3	8 33	Acc Rep 29			33		Acc Rep 13 14 14	Acc Rep 7 7 23 10 23	7 7	24

Table A23. Indices of Failing Labs Correlation Scores Atom Synthetic Oils

Atomic Absorption January 1985

Fe	Ag	AL	Cr	. Cu	Ng	\$i	Ti	, Mo	Ni -
Acc Rep 9 11 11 36 25 37 29 35 36 37	3 5	Acc Rep 11 19 31 32	Acc Rep 3 33 11 19 22 26 28 31 33 35 36 37 38			Acc Rep 13 20 20		Acc Rep 20	Acc Rep 16 16 28 28 31 34

Table A24. Indices of Failing Labs Proposed Scores Atom Synthetic Oils

Atomic Absorption January 1985

Fe	Ag	Al	Cr	Cu	Mg	Si	Ti	Mo	Ni
Acc Rep 9 29	10	Acc Rep 11 11 19		Acc Rep 11 36	Acc Rep 9 35	Acc Rep 13 20	Acc Rep 25	Acc Rep 20 20	Acc Rep 31 28

#### APPENDIX B

This appendix describes the mathematical computations used for the proposed scoring system. The same procedure is followed for each element, for each type of sample, for either type of instrument. As in the text above, let

$$x_i$$
 - Analysis result for sample 1, laboratory i  $y_i$  - Analysis result for sample 2, laboratory i.

The first step is to perform the bivariate trimming. Define

$$\bar{x} - \Sigma x_i/n$$
 - Average of all sample 1 results

$$\hat{y} - \Sigma y_i/n$$
 - Average of all sample 2 results

where n is the total number of instruments analyzing this sample. Now define

$$A = \Sigma(x_i - \bar{x})^2/(n - 1)$$

$$C = \Sigma(y_i - \bar{y})^2/(n - 1)$$

$$B = \Sigma(x_i - \bar{x})(y_i - \bar{y})/(n - 1)$$

and let S be the 2 by 2 matrix whose first row is A, B and whose second row is B, C; define T to be the matrix inverse of S. This matrix T is used to evaluate n quadratic forms, one for each participating laboratory. That is, for instrument i the quadratic form is

$$Q_i = t_{11}(x_i - \bar{x})^2 + 2t_{12}(x_i - \bar{x})(y_i - \bar{y}) + t_{22}(y_i - \bar{y})^2$$

where the first row of T is  $t_{11}$ ,  $t_{12}$  and the second row of T is  $t_{12}$ ,  $t_{22}$ . These  $Q_i$  values then are ranked in order of magnitude, from smallest to largest and are used to trim off (no more than) 5% of the instruments; if for example n = 183 instruments had analyzed the sample, 5% of n equals 9.15 so the 9 largest  $Q_i$  values identify those instruments to be trimmed off. The remaining 174 laboratories are used to determine the accuracy and repeatability limits. Let m represent the number of instruments remaining after trimming; m, of course, is the next larger integer above .95n (or .95n rounded up).

The trimmed means, for the x and y scores, are the averages of the m remaining pairs. Hopefully without confusion, let  $\tilde{x}$  and  $\tilde{y}$  represent these trimmed means and define

$$X_i - x_i - \tilde{x}$$

$$Y_1 - y_1 - \bar{y}$$

for all n instruments. The accuracy score for instrument i then is

$$A_i - (X_i + Y_i)/2.5$$

and the repeatability score for instrument i is

$$R_i = (X_i - Y_i)/2^{.5}$$
.

We now have n pairs,  $(A_i, R_i)$ , one for each instrument. Using m instrument pairs which were not trimmed initially, define the accuracy and repeatability standard deviations by

$$s_a = (\Sigma A_i^2/(m - 1))^{.5}$$

$$s_r = (\Sigma R_i^2/(m - 1))^{.5}$$
.

The accuracy limit is  $3s_a$  and the repeatability limit is  $3s_r$ ; any instrument, trimmed or not, whose accuracy score or repeatability score exceeds the respective limit in magnitude (absolute value) fails on that score.

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